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A NEW APPROACH - DERIVING EXPECTED FUTURE INCOME FOR INTERTEMPORAL MODELS

Abstract:

Individuals' current consumption decisions are influenced by the prospect of future income. This paper discusses a method for inferring agents' future (expected) income based on expectations of future states of the environment. Psychology, risk, the probability of future events occurring, etc. play an important role in this area. These essential parts have been incorporated into a simplified model that will make it easier for us to determine the value of future income and serve to more accurately incorporate agents' positive or negative expectations into intertemporal two-period or overlapping-generation models. This paper brings a new approach to this issue based on prospect theory and reference point theory. The issue is addressed only at the microeconomic level, while this approach can easily be aggregated into macroeconomic models.

Keywords:

probability of future, income, OLG models, two-period model, psychology, risk, expectation income

JEL Classification: D01, D90, D91

1 Introduction

The consumer's income is the alpha and omega of decision-making. Every agent can imagine what his consumption will be at his current or other income. For this reason, the agent does not plan his future at the consumption level, but according to his expected future income, which precedes the consumption or savings itself.

Decisions about current levels of consumption, debt or savings (or investment) are based on current and future (expected) income. The determinants of future income based on these assumptions will be:

- the present value of the pension;
- the expectation of the probability of survival, which is given by the probability of death function;
- expectations about gains and losses arising from expected negative and positive states of the environment;
- expectation of extra income unrelated to your current pension (lottery winnings, inheritance, etc.).

Given the value of current income, which is the agent's benchmark, cumulative prospect theory (Tversky and Kahneman 1992) seems ideal for applying valuation of future income, or future (expected) gains and losses, to current income. In our case, however, there will be some modification, since cumulative prospect theory is based on the agent's decision about the values of different paths, while the circumstances that will affect the agent's future income do not contain a decision element, since they are exogenous. For example, an increase in the probability of bankruptcy of the firm in which the agent works is a phenomenon that is not decided by the agent because it is the result of exogenous phenomena (exogenous to the agent).

In our analysis, we will therefore assume that the agent works with the expectation of income and consumption as follows: in the first step, he knows his current income, which has a probability value of 1 from the perspective of the present moment, so it is certain. Based on future different states of the environment, he expects whether his current income will decrease, stagnate or increase. There are a number of endogenous or exogenous factors that affect the expectation of future disposable income, e.g.: developments in the world economy, developments in the domestic economy, the political situation, changes in the labour market (changes in employment and unemployment), etc. At the next stage, only the agent will make decisions about his current consumption and only at the final stage will he make decisions about his future consumption.

This work is based on the results of the dissertation of Jiří Rotschedl (Rotschedl, 2022a). Its results have been applied in papers dealing with agents' intertemporal decision making by age (Rotschedl and Mitwally, 2021) or by savings and debt (Rotschedl, 2022b).

Expectations about future income is one area that is key at the macro level but also at the micro level. While some papers extend the decision-making to other areas such as transportation costs (Kucera and Kaderabkova, 2023), these studies focus on the other side of the same coin, namely expected costs. One can intuitively expect that if an agent expects a negative development (e.g., he expects his income to stagnate or be threatened), transaction costs will be among the first costs that the agent will stop accepting and will most likely exclude from consumption.

These findings merit further detailed investigation and study.

In the period 2020-2022, we have experienced COVID. This phenomenon affected every agent in the economy, and it can be concluded that COVID was among the key factors that greatly affected future expected income. Governments tried to eliminate the risk through various programs (Zubikova, Veselá, Smolák, 2023) to save jobs and keep the economy stable despite lockdowns. In the Czech Republic, these programs were very generous, contributing to accelerating inflation in 2022 and 2023.

2 Factors affecting future disposable income

Factors influencing disposable income were surveyed by ING Bank (2013) among approximately 17,000 respondents across Europe and include:

- price increase
- loss of employment
- incurring unplanned expenditure
- salary reduction

A self-reported survey[1] (December 2015 - January 2016) was conducted among respondents (n = 87, 55% female, 45% male) regarding their expected retirement, and the ranking of the factors rated by respondents as most important from most important is as follows:

- Self-inflicted ($\bar{x} = 3.644$, $\sigma^2 = 2.068$)
- Employment situation ($\bar{x} = 3.494$, $\sigma^2 = 2.135$)
- The situation in the entire industry in which the company operates ($\bar{x} = 3.379$, $\sigma^2 = 1.362$)
- Medical condition or age ($\bar{x} = 3.333$, $\sigma^2 = 1.900$)
- Development of the domestic economy ($\bar{x} = 2.805$, $\sigma^2 = 1.628$)
- Price changes ($\bar{x} = 2.69$, $\sigma^2 = 1.984$)
- Labour market situation - employment trends ($\bar{x} = 2.678$, $\sigma^2 = 2.126$)
- Labour market situation - unemployment trend ($\bar{x} = 2.563$, $\sigma^2 = 2.292$)
- Economic developments in the world ($\bar{x} = 2.517$, $\sigma^2 = 1.629$)
- The political situation ($\bar{x} = 2.287$, $\sigma^2 = 2.044$)
- Changes in interest ($\bar{x} = 2.069$, $\sigma^2 = 1.949$)
- The mood in society ($\bar{x} = 2.023$, $\sigma^2 = 1.953$)

Values below 2.5 represent factors that, according to the survey, do not have a significant impact on future pension. Among these factors, there is only one that is endogenous, i.e. the future pension is influenced by the agent himself (his effort, diligence, laziness, etc.). The other factors are exogenous, which cannot be influenced by the agent himself because he does not have the economic or political power to do so.

In the following, these factors will represent the agent's **environment**. These environments can have different states. For example, the "price change" neighbourhood can take several states, e.g.: they will increase by 1%, by 3%, or they can stagnate or decrease by 1.5%. Each state has a certain probability, which will be represented by weights (the sum of the probabilities of all states is equal to one). In addition to the probability of the environmental states and the calculation of the value of the expected change in disposable income, it is possible to determine a risk measure (weighted by

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[1] Respondents rated on a scale of 0-5 the importance of the criteria that determine their future retirement.

the standard deviation) that will not be decisive for the purposes of calculating the expected income. The general form of the equation for calculating the expected future income arising from the j[th] environment is:

$$I_{t+1}^{j} = I_t \,.\sum_{i=1}^{v}\left(1 + k_i^{j}\right).p_i^{j} \qquad (1)$$

Under the conditions: $\sum_{i=1}^{v} p_i^{j} = 1$

Where:

I_{t+1}^{j} = future (expected) income related to the jth environment

I_t = current (certain) pension

k_i^{j} = the rate of change of future income relative to the current income induced in relation to the j-th environment and the i-th state of this environment

p_i^{j} = weights, i.e. the subjective probability of the i-th state of the j-th environment, which are calculated from the objective probability π_i^{j} and a function accounting for risk aversion and loss aversion.

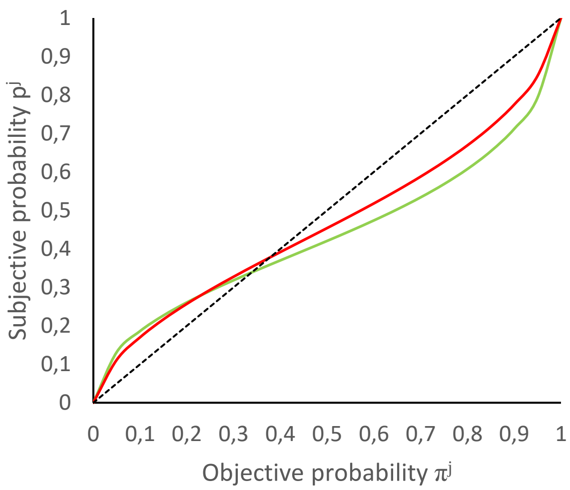
In the models, a relationship based on the Tversky and Kahneman (1992) function can be used to calculate the subjective probability:

$$p_i = \frac{\pi_i^{\gamma}}{\left(\pi_i^{\gamma} + (1-\pi_i^{\gamma})\right)^{1/\gamma}} \qquad (2)$$

Where γ denotes a coefficient expressing the relationship to risk or loss. Based on empirical data, the values were determined: $\gamma^{+} = 0.61$ (for profits) and $\gamma^{-} = 0.69$ (for losses) (see Tversky and Kahneman, 1992). The form of the relationship between subjective and objective probability is shown in the following graph. The function, originally derived in 1979 by Kahneman, was later (in 1992) refined by Tversky using cumulative prospect theory.

It is clear from the graph that we are more willing to take risks for events that have low probability (we are risk averse), which explains the willingness of agents to bet in the lottery. The different waveforms of the functions correspond to a lower risk aversion for losses (or willingness to take risks), which is explained by loss aversion.

Chart 1: Graphical representation of the relationship between subjective and objective probability



Source: own elaboration according to Tversky, Kahneman 1992

If it exists in ***the*** neighborhood, then the total value of expected future income I_{t+1} can be calculated:

$$I_{t+1} = I_t \,.\prod_{j=1}^{u}\sum_{i=1}^{v}\left(1+k_i^j\right).p_i^j \qquad (3)$$

For the sake of simplicity, we adapt the model to only two basic states of the jth neighborhood: **good (g)** - **bad (b)** with probability p_g^j a p_b^j, with $p_g^j + p_b^j = 1$. The probabilities of the negative and positive states of the environment can be expressed in terms of a single probability. The neighborhood state **good** will represent the best possible variant and **bad will represent** the worst possible state of the neighborhood, or the reference interval.

A simplified equation for calculating the expected future income in different jth neighborhoods of an agent that can have good or bad state will be as follows:

$$I_{t+1} = I_t \,.\prod_{j=1}^{u}\left[\left(1+k_g^j\right).p_g^j + \left(1+k_b^j\right).(1-p_g^j)\right] \quad (4)$$

The agent's future expected pension at the microeconomic level excluding induced pension [1]I_{t+1} can expect an autonomous future pension I_{t+1}^Awhich represents the value of the extra income from winning the lottery, receiving an inheritance, etc. Autonomous income will not be considered at the aggregate level, since for some people a lottery win is an expense and for others an income, i.e. in aggregate form, future expected income can be derived from agents' current income.

Based on equation (4), there are 2 uncertain components: at what rate the agent's income will grow/decline (relative to current income) and with what probability these states will occur. Different circumstances affecting future pension may have different significance. For example, for an agent with scarce human capital (e.g., a statistician), the threat of job loss will be lower than for an agent with easily substitutable human capital (an administrative worker). In such a case, an increase in unemployment increases the risk of being laid off, i.e., the probability of a negative environmental condition increases, which leads to a reduction in the expected future pension of the agent with low skills but may have little effect on the expected pension of the agent with a specialization that provides stable employment even in times of economic crisis. Equation 4 will therefore be adjusted for the weights of the importance of each neighbourhood on the agent's future pension:

$$I_{t+1} = I_t \,.\prod_{j=1}^{u}\left[\left(1+k_g^j\right).p_g^j + \left(1+k_b^j\right).(1-p_g^j)\right]^{\mathrm{w}^j} \qquad (5)$$

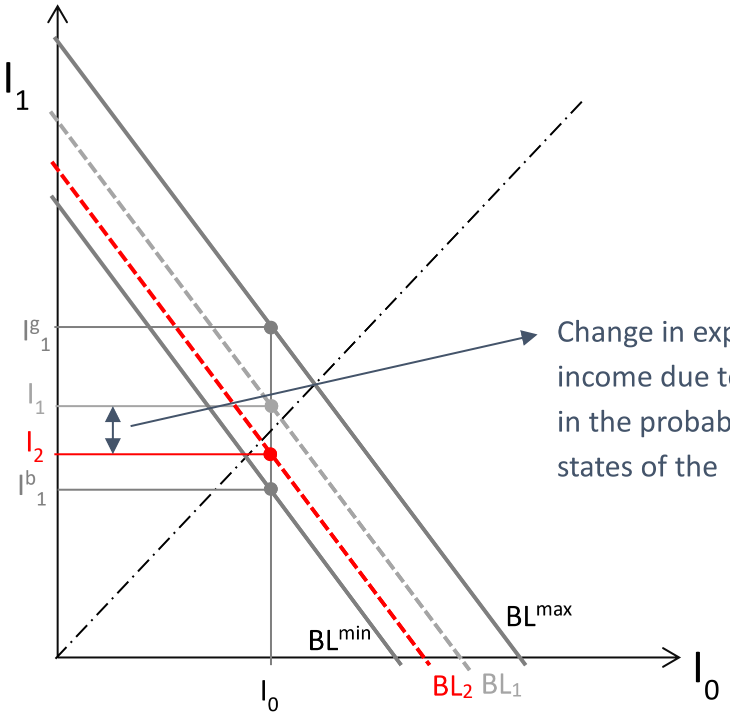
Where w^j represents the weights and it must be the case that $\sum_{j=1}^{u} w^j = 1$.

Thus, on the one hand, agents receive an income, which in the intertemporal sense is an endowment, but which may not suit the agents, resulting in negative or positive savings that allow a better allocation of consumption within the present moment and the expected future.

The plotting of uncertain income is demonstrated by the following graph. The uncertainty of future income will shift the agent's budget constraint.

[1] The concept of induced pension is meant here as a pension dependent on a previous pension.

Chart 2 - Graphical representation of the change in probability and impact on the intertemporal budgetary constraint



Source: own processing

The derivation of future pension will vary for each agent in particular:

- the relationship to risk for gains and losses (the agent is willing to take more risk for losses);
- the importance of the environment relative to expected income;
- of each agent's survival probability.

3 Probability of survival - inference on Czech Republic data

The survival probability function, or its inverse form (the probability of death function) is compiled for the Czech Republic by the Czech Statistical Office at the general level. The shape of the probability of death function according to the CZSO at age x is as follows:

$$g_x = 1 - e^{-m_x} \quad (6)$$

whereby, m_x is a specific rate that is equal to the ratio of deaths at a given age to the median population (m_x = D /P_{xx}). According to the CSO methodology, the values in each group are smoothed. Data from the CSO will be used to calibrate the parameters of our own model, which introduces two parameters that will be used to simulate the effect of long-term or fatal diseases, or the quality of life in each economy, in the survival function.

The modified equation has the following form:

$$E_t = 1 - [(\delta - 0{,}003m) - v]^{n\prime-(l+t)} \quad (7)$$

where ***n' = n - m***, where ***n*** is the theoretical maximum life expectancy in the society under study, based on empirical data, and ***m*** expresses the reduction in maximum life expectancy following illness or injury. In the Czech data, n = 110 for men and n = 107 for women. In the Czech Republic, although women generally live to a higher age than men, men are more likely to live to a higher age than women of the same age if they live to 70.

This equation has the following additional parameters:

- $\boldsymbol{\delta}$: we will consider the average quality of life, health care, environment, food quality, as well as morbidity, negative habits in society. Value $\delta \in < 0; 1 >$.
- Value δ thus represents a parameter that talks mainly about the overall quality of life in a given society/country. The lower the value, the better the quality of life in old age, or the steeper the curvature of the survival probability curve, which means that at older ages people have a higher probability of surviving to a given age than, for example, their peers who live in an environment for which the value applies δ higher.
- $\boldsymbol{\upsilon}$: expresses the adjustment of the probability of survival, either negatively (in case of disease) or positively (by starting disease prevention). If the area is negative, then the parameter values reflect different rates of disease deterioration, with the following $\upsilon \in < 1 - (\delta - 0{,}003m);\ \delta - 0{,}003m >$where:
- $\boldsymbol{\upsilon = \delta - 0{,}003m}$ The parameter expresses sudden death (fatal accident or sudden fatal illness). The resulting survival probability is: $E_t = 0$.
- $\boldsymbol{\upsilon \in (0; \delta - 0{,}003m})$ a parameter in this interval expresses a higher probability of survival than the average inhabitant, i.e. a person who focuses significantly on disease prevention, prefers a healthy lifestyle, etc.
- $\boldsymbol{\upsilon = 0}$ represents the average sick person, with an average propensity to prevent corresponding to the population on which the model was calibrated.
- $\boldsymbol{\upsilon \in (1 - (\delta - 0{,}003m);\ 0})$ expresses the interval of values corresponding to the rate of deterioration of the disease. The lower the value, the faster the disease progresses.

The following survival probability models were derived for the Czech Republic for average males and females based on CSO data:

Men (2013): $$E_t = 1 - [(0{,}92 - 0{,}003m) - \upsilon]^{(110-m)-(l+t)} \quad (8)$$

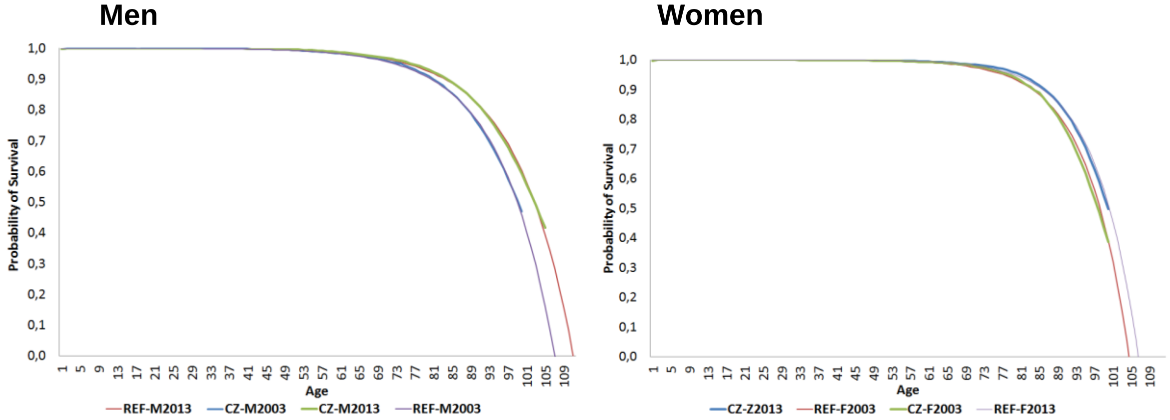
Women (2013): $$E_t = 1 - [(0{,}895 - 0{,}003m) - \upsilon]^{(105{,}5-m)-(l+t)} \quad (9)$$

For comparison in 2003, the values in the equation for men in the Czech Republic would be: δ= 0.918 and *n* = 106 (e.g. in Hungary: δ = 0.905, *n* = 103; in Slovakia: δ = 0.927, *n* = 109; in Austria and Sweden identically: δ = 0,918, *n* = 109).

Chart no. 3 presents the real data on which the models in equations (28) and (29) were calibrated.

Based on the parameters $\boldsymbol{\delta}$ and ***n***, it is possible to compare countries or even smaller territorial units with each other or with developments over time. These two variables cannot be considered separately. Both values contribute to the shape of the survival probability function not only by their length but also by their roundness.

Chart no. 3 Comparison of actual and modelled survival probability curves in the Czech Republic, 2003 and 2013



source: own processing from CZSO data

Chart no. 3 indicates that both sexes experienced an increase in life expectancy between 2003 and 2013, but that men experienced a slight decrease in quality of life, while women experienced a slight increase. The thicker curves represent real data according to the CSO and the thin curves (REF-M/F2013, REF-M/F2003) represent probabilities calculated according to the derived model, see equation (7).

The resulting equation for calculating expected future income after taking into account the probability of survival and autonomous income will therefore take the form:

$$I_{t+1} = E_{t+1}.\left\{ I_t \; .\prod_{j=1}^{u}\left[\left(1+k_g^j\right).p_g^j + \left(1+k_b^j\right).(1-\,p_g^j)\right]^{w^j} + \sum_{h=1}^{x} I_h^A \,.p_h^A \right\} \qquad (10)$$

Under the conditions: $\sum_{j=1}^{u} w^j \; = 1$

Where

E_{t+1} = probability of survival at time t+1;

I_h^A = h-th autonomous income of the agent (lottery winnings, inheritance, etc.);

p_h^A = hth subjective probability of occurrence I_h^A retirement;

w^j = weights of individual states of the environment ***j***.

Thus, the expected budget constraint of each agent is given by the current and expected future income, which can be expressed as the value of uncertain scenarios (states of the environment) that may occur in the future in the agent's neighbourhood or are admissible. In the text above, we have tried to derive a calculation of expected income that considers new insights about agents' risk behaviour. Given reference-dependent decision making, it can be assumed that an agent's future pension will always be derived from a reference pension, which is the current (certain) pension.

The issue of interest rates impacting an agent's disposable income has not yet been discussed. In practice, we encounter two levels of interest rates - the income interest rate on savings (r_S) and the cost interest rate on loans (r_L). The effect of interest rates on consumption formation is debatable, see e.g. Bernanke and Gretler (1995) or Rotschedl (2014), and the future interest rate also exhibits uncertainty, as the future path of income is as uncertain as the path of the interest rate and therefore the return on savings at t = 0 can be embedded as one neighbourhood of the agent, or the state of this neighbourhood.

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